MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

26 May 2017
3.00p.m – 5.00p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carry equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.

Answer All Questions.

Question 1 (10 marks)

a) Given $f(x)=\begin{cases} x^2+4x, & x\le -2 \\ \dfrac{x+2}{x^2-4}, & -2<x<2 \end{cases}$.

Find i) $\lim_{x\to 3} f(x)$ (1 mark)

ii) $\lim_{x\to -2^+} f(x)$ (2 marks)

iii) $\lim_{x\to -2} f(x)$ (2 marks)

Show that

iv) the function $f(x)$ is continuous at $x=-4$. (2 marks)

b) Find $\lim_{x\to -1}\dfrac{\sqrt{x^2+24}-5}{x+1}$. (3 marks)

Question 2 (10 marks)

a) Find the first derivative given:

i) $y=\dfrac{3x^2+4x-10}{2x}$. (2 marks)

ii) $y=\sin^5(5x^2+\pi x)$. (3 marks)

b) Find the equation of the tangent line to the curve $y=x^2\ln(2e-x)$ at the point $x=e$. (5 marks)

Question 3 (10 marks)

Given $f(x)=\dfrac{x^4}{4}-2x^2+4$.

a) Find the local maximum, local minimum, interval of increasing and interval of decreasing. (4 marks)

b) Find the inflection points, interval of concave up and interval of concave down. (4 marks)

c) Sketch the graph of $f(x)$. (2 marks)

Continued...

Question 4 (10 marks)

a) Solve the given first order differential equation, $xy'+2y = x^2 - x + 1$, $y(1) = 0$. (4 marks)

b) Solve the given second order differential equation.
$y''+4y'+5y = 0$, $y(0) = 1, y'(0) = 0$ (6 marks)

Question 5 (10 marks)

a) Evaluate:

i. $\int_0^2 [(x+4)^2 + \cos \pi x]dx$ (3 marks)

ii. $\int (x+2)e^{x^2+4x} dx$ (2 marks)

b) Use integration by partial fraction given $\int \frac{x+3}{x^3 - 25x^2} dx$ (5 marks)

End of Paper

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$